Int. J. Heat Mass Transfer. Vol. 5, pp. 1191–1205. Pergamon Press 1962. Printed in Great Britain.

THE PREDICTION OF HEAT AND MASS TRANSFER COEFFICIENTS FOR TURBULENT FLOW IN PIPES AT ALL VALUES OF THE PRANDTL OR SCHMIDT NUMBER

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(*Received* 5 *June* 1962)

Abstract—A simple model of turbulent heat or mass transfer based on a modified form of the Reynolds analogy, is proposed. Equations have been derived from which heat or mass transfer coefficients, and temperature or concentration profiles, may be predicted at any value of N_{Pr} or N_{Sc}. In deriving the equations it is assumed (i) that, in pipe flow, the transport mechanism is such that there is no significant molecular transport in the turbulent core even at low N_{Pr}, and (ii) that eddy transport is a function of the flow pattern only. The equation is of the form

$$N_{St} \text{ (or } N_{Sh}) = \frac{f/2}{\varphi}$$

and φ is given both in algebraic form and as a plot of φ versus N_{Pr} (or N_{Sc}) for smooth pipes.

Computations of N'_{Nu} and temperature profiles agree well with experimental data. Particular attention is given to results at low N_{Pr}, where the assumptions made as to transport in the turbulent core have the greatest effect; in this region the proposed equations predict experimental results more closely than do other correlations. In the intermediate range of N_{Pr} or N_{Sc} the proposed equations agree with other analyses, and at high values of N_{Sc} the equations reduce to those of Lin *et al.* [1] which are in excellent agreement with experimental data.

NOMENCLATURE

A, area for heat transfer, ft^2, at pipe wall, A_r at radial distance r from the centre;
C, time average concentration lb moles/ft^3; C_b, mixed mean concentration; C_W, concentration at pipe wall;
Cp, specific heat Btu/lb degF;
D, molecular diffusivity of mass, ft^2/h;
f, fanning friction factor;
$f(Z)$, function of Z; $f(Z)_M$, function of Z for momentum transfer; $f(Z)_H$, function of Z for heat transfer; $f(Z)_D$, function of Z for mass transfer;
h, heat transfer coefficient, Btu/h ft^2 degF;
k, thermal conductivity Btu/h ft degF;
k_C, mass transfer coefficient, ft/h;
N, mass transfer, lb moles/h ft^2, at wall, Nr at radial distance r from the centre;
N'_{Nu}, Nusselt number $= (h2R)/k$ or equivalent mass transfer group $(k_C 2R)/D$;
N'_{Pe}, Peclet number $= N_{Re} \times N_{Pr}$ or equivalent mass transfer group $N_{Re} \times N_{Sc}$;
N_{Pr}, Prandtl number $= (Cp\mu)/k$;
N_{Re}, Reynolds number $= (\rho u_b 2R)/\mu$;
N_{Sc}, Schmidt number $= \mu/\rho D$;
N_{St}, Stanton number $= h/(\rho C p u_b)$;
N_{Sh}, Sherwood number $= k_C/u_b$;
q, heat load, Btu/h, q_W heat load at wall;
R, radius of pipe, ft;
R^+, dimensionless radius of pipe Ru^*/ν;
r, distance from centre of pipe, ft;
t, time averaged temperature, degF; t_b, mixed mean temperature; t_B, temperature at $y^+ = 33$; t_C, temperature at $Z = 0{\cdot}8$; t_L, temperature at $y^+ = 5$; t_M, temperature at $y^+ = 100$; t_W, temperature at wall;
u, time average axial velocity at any radial position ft/h; u_b, mean velocity; u_B, velocity at $y^+ = 33$; u_C, velocity at

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$Z = 0{\cdot}8$; u_M = velocity at $y^+ = 100$; u_0, velocity at centre of pipe;
u^*, friction axial velocity ft/h $u_b\sqrt{(f/2)}$;
u^+, dimensionless axial point velocity u/u^*;
U, dimensionless axial point velocity u/u_b;
y, distance from pipe wall, ft;
y^+, dimensionless distance from pipe wall, yu^*/ν;
Z, dimensionless distance from the centre of the pipe, r/R;
α, thermal diffusivity, ft²/h;
ϵ, eddy diffusivity, ft²/h; ϵ_M, eddy diffusivity of momentum; ϵ_H, eddy diffusivity of heat; ϵ_D, eddy diffusivity of mass;
μ, viscosity, lb/ft h;
ν, kinematic viscosity, ft²/h;
ρ, density, lb/ft³;
τ, shear stress, lb/ft² h², at wall, τ_r at radial distance r from the centre.

INTRODUCTION

It is possible to predict heat and mass transfer coefficients, and the corresponding profiles, by analogy with momentum transport if a relation between the transport processes is known or assumed. Reynolds' original assumption of complete identity of heat and momentum transfer results in the relation $N_{St} = f/2$, which is only true for $N_{Pr} = 1$, and later workers have made more elaborate analogies to provide equations of wider utility. These relations can be expressed as $N_{St}\varphi = f/2$ where $\varphi = \psi(N_{Pr}, f/2)$. Recent equations have generally been developed to agree with experimental data in some definite N_{Pr} or N_{Sc} range and of these the equations for heat transfer in liquid metals and for mass transfer in liquids are of particular interest. In the liquid metal range the noteworthy relations are: Martinelli's [2] equation for constant heat flux, Lyon's [3] simplified equation for the same condition, Seban and Shimazaki's [4] equation for constant wall temperature and variable radial heat flux, Deissler's [5] and Azer and Chao's [6] equations for variable ϵ_H/ϵ_M. For mass transfer in liquids only the equations of Lin *et al.* [1] and of Deissler [7] are sufficiently accurate to merit consideration. Some of these equations may be extended with reasonable accuracy over a wide range of N_{Pr} or N_{Sc} but none is universal in its application. In this paper an equation is developed which may be used to evaluate heat or mass transfer profiles and the corresponding coefficients over the entire N_{Pr} or N_{Sc} range and at all turbulent N_{Re}.

In deriving this equation only incompressible fluids in fully developed turbulent flow in circular pipes are considered and it is assumed that constant flux, steady state conditions apply so that transfer coefficients are independent of pipe length.

Defining the eddy diffusivities by the equations

$$(\epsilon_M + \nu)\frac{du}{dy} = \frac{\tau_r}{\rho} = (u^*)^2 \, . f(Z)_M \qquad (1)$$

$$(\epsilon_H + \alpha)\frac{dt}{dy} = \frac{q_r}{A_r \rho Cp} = \frac{q}{A\rho Cp} \, . f(Z)_H \qquad (2)$$

$$(\epsilon_D + D)\frac{dC}{dy} = N_r = N \, . f(Z)_D \qquad (3)$$

the temperature or concentration profiles may be obtained by substituting the relations of the following section into equations (2) or (3).

VELOCITY PROFILE

The velocity profile has been divided into four regions. The equations for the first two of these are due to Lin *et al.* who modified von Kármán's universal profile by introducing an eddy of magnitude $\epsilon_M/\nu = (y^+/14{\cdot}5)^3$ into the laminar layer. This agrees with the general considerations of Hinze [8] and gives good agreement with experimental mass transfer coefficients. The equations employed for the first two regions are therefore:

for $0 \leqslant y^+ \leqslant 5$ (laminar layer)

$$u^+ = \frac{14{\cdot}5}{3}\left\{\frac{1}{2}\ln\frac{[1 + (y^+/14{\cdot}5)]^2}{1 - (y^+/14{\cdot}5) + (y^+/14{\cdot}5)^2} + \sqrt{3}\tan\frac{(2y^+/14{\cdot}5) - 1}{\sqrt{3}} + \frac{\sqrt{3}}{6}\right\} \qquad (4)$$

and for $5 \leqslant y^+ \leqslant 33$ (transition region)

$$u^+ = 4{\cdot}77 + 5\ln\left(\frac{y^+}{5} + 0{\cdot}041\right) \qquad (5)$$

and

$$\frac{\epsilon_M}{\nu} = \frac{y^+}{5} - 0{\cdot}959. \qquad (6)$$

For the third region, $33 \leqslant y^+ \leqslant R^+/5$

$$u^+ = 5{\cdot}5 + 2{\cdot}5 \ln y^+ \tag{7}$$

and

$$\epsilon_M = \frac{Ru^*}{2{\cdot}5}\left(\frac{y}{R}\right)\left(1 - \frac{y}{R}\right). \tag{8}$$

For the central core, $y^+ \geqslant R^+/5$, the best fit with experimental data is given by [8]

$$u^+ = \frac{u_0}{u_b} - 7{\cdot}2\, Z^2 \tag{9}$$

and

$$\epsilon_M = \frac{Ru^*}{14{\cdot}4} \tag{10}$$

where [9] $u_0 = 4{\cdot}25\, u^* + u_b$.

TEMPERATURE PROFILE

Dividing equation (2) by equation (1) gives

$$\mathrm{d}t = \frac{q}{A\rho Cp} \cdot \frac{1}{u^*} \frac{\epsilon_M + \nu}{\epsilon_H + \alpha} \cdot \mathrm{d}u^+ \cdot \frac{f(Z)_H}{f(Z)_M}. \tag{11}$$

This equation must be integrated for the various regions into which the profile is divided and to do this various assumptions are made and these are now discussed:

(a) $R^+/5 \leqslant y^+ \leqslant R^+$

It is assumed for this parabolic velocity distribution region that all transport takes place by eddies and that $\epsilon_H = \epsilon_M$ so that

$$(\epsilon_M + \nu)/(\epsilon_H + \alpha) = 1.$$

In addition, the ratio $f(Z)_H/f(Z)_M$ has been equated to unity. [See Appendix I].

On this basis equation (11) can be integrated to give

$$t = \frac{q}{A\rho Cp} \cdot \frac{1}{u^*} \cdot u^+ + \text{const.} \tag{12}$$

which was then checked by comparison with experimental data in Figs. 1 and 2. These show that the slope of the line t versus u^+ can be reasonably represented by $(q/A\rho Cp) \cdot (1/u^*)$ in accordance with equation (12) thus supporting the assumptions made. Finally, calculating values of ϵ_H on the above basis from recent data [10] for ϵ_H and ϵ_M for turbulent flow of mercury gives qualitative agreement with the present assumptions and good agreement for

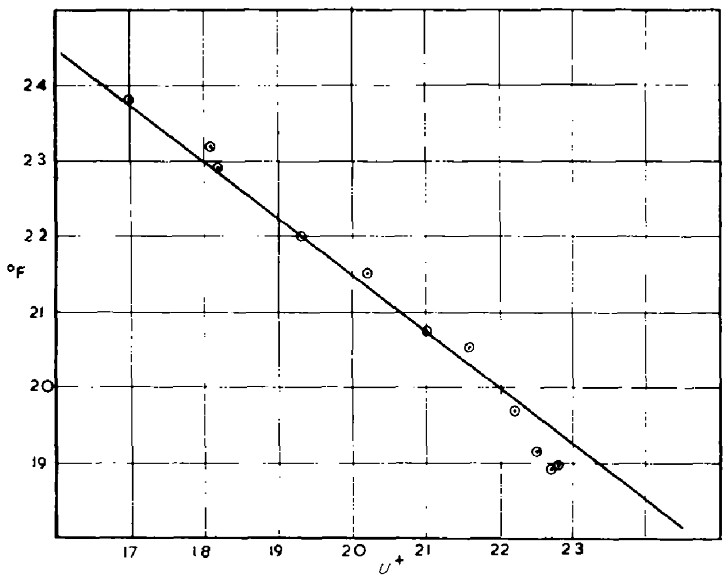

FIG. 1. Comparison of measured values of temperature and velocity in mercury by Isakoff [11] with calculated values,

○ Isakoff $N_{Re} = 3{\cdot}76 \times 10^4$ — Calculated.

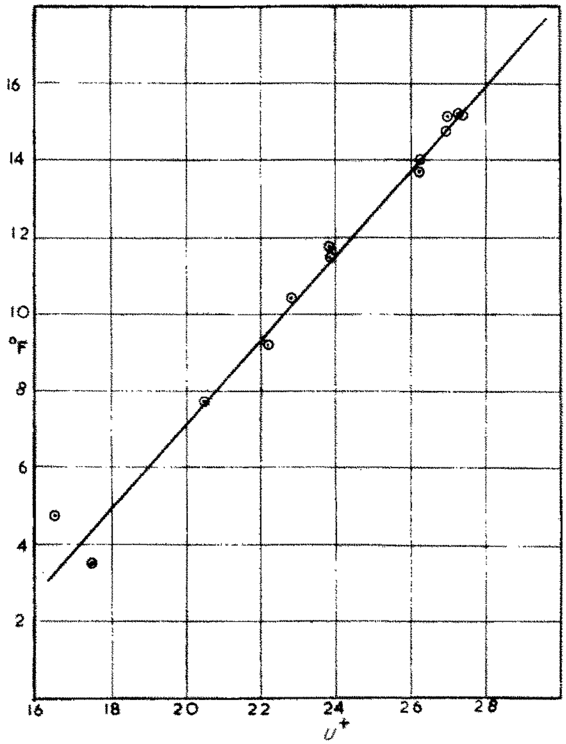

FIG. 2. Comparison of measured values of temperature and velocity in mercury by Brown *et al.* [14] with calculated values,

⊙ Brown *et al.* $N_{Re} = 66 \times 10^4$
— Calculated.

heat transfer coefficients. The small discrepancies involved may be due to the fact that the authors [10] apparently made no allowance for possible transverse eddies in their rectangular duct.

(b) $100 \leqslant y^+ \leqslant R^+/5$

The limiting position at which molecular transport of heat ceases to have any significant value is assumed to be at $y^+ = 100$. (This value is taken from a consideration of the temperature profiles at low N_{Pr}.) As a result the logarithmic velocity distribution region has been considered as made up of two parts in the calculation of temperature profiles. The assumptions in this region are the same as in the region $R^+/5 \leqslant y^+ \leqslant R^+$ above, again giving equation (12) as the integrated form of equation (11).

(c) $33 \leqslant y^+ \leqslant 100$

In this region since α is about ten times as great as ϵ_H and ν is about one tenth the value of ϵ_M, ν is neglected in equation (11). Again it is assumed that $\epsilon_H = \epsilon_M$, that ϵ_H and α are additive, and that $f(Z)_H = f(Z)_M$ which is true for this region close to the wall.

(d) $0 \leqslant y^+ \leqslant 33$

For the laminar and buffer regions it is again assumed that $f(Z)_H = f(Z)_M$ and that ϵ_M and ν and ϵ_H and α are scaler additive. As stated in the previous section the value of the eddy assumed by Lin *et al.* is adopted here.

The equations based on the above assumptions apply strictly for $N_{Re} \geqslant 4 \times 10^4$ as there are no experimental profiles available below this N_{Re} for low N_{Pr} values. For $N_{Re} < 4 \times 10^4$ the temperature profiles have been predicted by retaining the assumption that the limiting value at which molecular transport has any effect is at $y^+ = 100$. The significance of this assumption is discussed in the section on the comparison of theoretical and experimental data.

The resulting equations which give the temperature profiles are:

(a) For $0 \leqslant y^+ \leqslant 5$

$$(t_W - t) = \frac{q}{A\rho Cp}\,\frac{1}{u^*}\,\frac{14{\cdot}5}{3}\,N_{Pr}{}^{2/3}\left(\left\{\tfrac{1}{2}\ln\frac{[1 + N_{Pr}(y^+/14{\cdot}5)]^2}{1 - N_{Pr}{}^{1/3}(y^+/14{\cdot}5) + N_{Pr}{}^{2/3}(y^+/14{\cdot}5)^2}\right\} + (\sqrt{3})\tan^{-1}\left\{\frac{N_{Pr}{}^{1/3}(2y^+/14{\cdot}5) - 1}{\sqrt{3}}\right\} + \frac{(\sqrt{3})\pi}{6}\right). \qquad (13)$$

(b) For $5 \leqslant y^+ \leqslant 33$

$$(t_L - t) = \frac{q}{A\rho Cp}\,\frac{5}{u^*}\ln\frac{1 + N_{Pr}[(y^+/5) - 0{\cdot}959]}{1 + 0{\cdot}041 N_{Pr}}. \qquad (14)$$

(c) For $33 \leqslant y^+ \leqslant 100$

$$(t_B - t) = \frac{q}{A\rho Cp}\frac{2\cdot 5}{u^*}\frac{1}{2 + N_{Pe}[\sqrt{(f/2)}/5]}\left(\ln\frac{y^+ - R^+ - (2\cdot 5/N_{Pr})}{33 - R^+ - (2\cdot 5/N_{Pr})} + \{1 + N_{Pe}[\sqrt{(f/2)}/5]\}\ln\frac{y^+ + (2\cdot 5/N_{Pr})}{33 + (2\cdot 5/N_{Pr})}\right). \quad (15)$$

But as the first term is always small this equation can be reduced to

$$(t_B - t) = \frac{q}{A\rho Cp}\frac{2\cdot 5}{u^*}\frac{1 + N_{Pe}[\sqrt{(f/2)}/5]}{2 + N_{Pe}[\sqrt{(f/2)}/5]}\ln\frac{y^+ + (2\cdot 5/N_{Pr})}{33 + (2\cdot 5/N_{Pr})}. \quad (16)$$

(d) For $100 \leqslant y^+ \leqslant R^+/5$

$$(t_M - t) = \frac{q}{A\rho Cp}\frac{2\cdot 5}{u^*}\ln\frac{y^+}{100}. \quad (17)$$

(e) For $R^+/5 \leqslant y^+ \leqslant R^+$ i.e. $0\cdot 8 \geqslant Z \geqslant 0$

$$(t_C - t) = \frac{q}{A\rho Cp} \times \frac{7\cdot 2}{u^*}(0\cdot 64 - Z^2). \quad (18)$$

The profiles calculated from these equations for low N_{Pr} are given in Figs. 3–6 where they are compared with experimental data. It will be seen that for the higher N_{Re} the predicted temperature difference is only about 4 per cent greater than that obtained experimentally. The experimental data taken from Isakoff [11] have been calculated by taking the tube-wall temperature as equal to the fluid-wall temperature, on the assumption of a negligible contact resistance. This was done because of obvious discrepancies in the original data, and the re-calculated results, which are about 30 per cent lower than the original, then agree with other experimental work. The assumption of negligible contact resistance is supported by the data of Mizushina *et al.* [12] on the contact conductivity of stagnant mercury which indicate that a correction for contact resistance would only become significant at $N_{Pe} > 20\,000$.

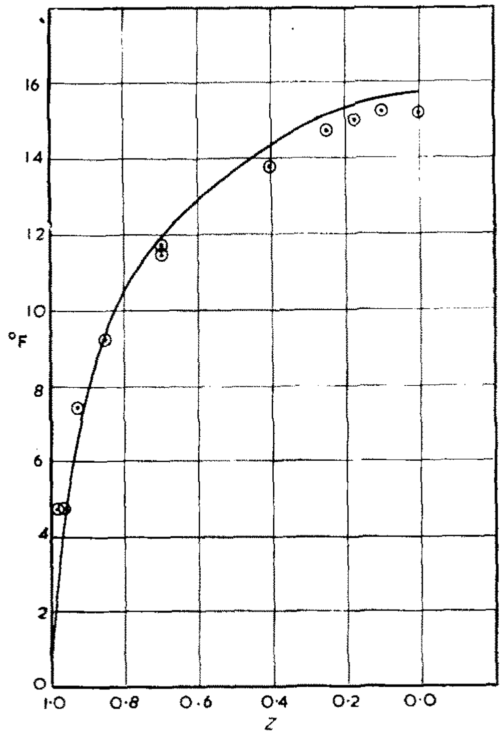

FIG. 3. Comparison of measured temperature distribution in mercury by Brown *et al.* [14] with calculated profile,

⊙ Brown *et al.* $N_{Re} = 66 \times 10^4$
— Calculated.

The dangers of using normalized profiles have been pointed out by Sleicher [13] but despite this many published results are still represented in this way. For example, Azer and Chao's normalized profiles show good agreement with both Isakoff's uncorrected data and with Brown's data [14] at a $N_{Re} = 350\,000$, although the corresponding values of N_{Nu} are 50 and 30 respectively. The use of a non-normalized profile would have clearly shown that there is in fact a considerable difference in the profiles at the different N_{Nu}. The data of Fig. 3 are shown again in Fig. 7 on a normalized basis to indicate the great loss in sensitivity which results from this type of plot.

Profiles are only shown for low N_{Pr} where the assumptions made in this section have the greatest effect.

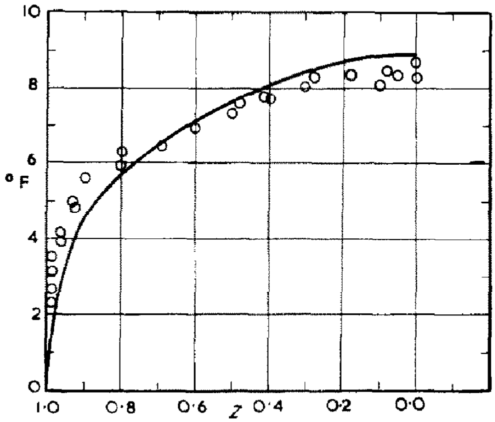

FIG. 4. Comparison of measured temperature distribution in mercury by Isakoff [11] with calculated profile,

○ Isakoff $N_{Re} = 3{\cdot}76 \times 10^4$
— Calculated.

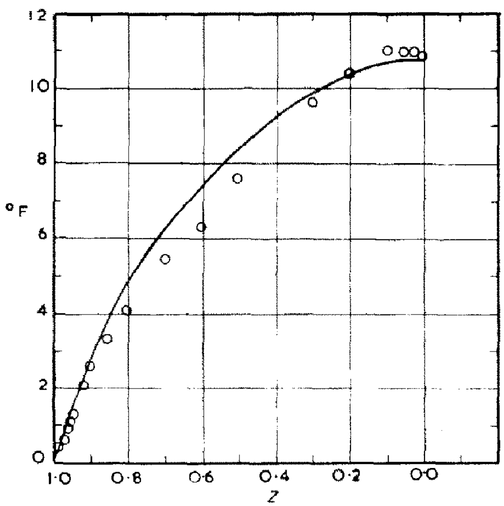

FIG. 5. Comparison of measured temperature distribution in mercury by Isakoff [11] with calculated profile,

○ Isakoff $N_{Re} = 4{\cdot}81 \times 10^4$
— Calculated.

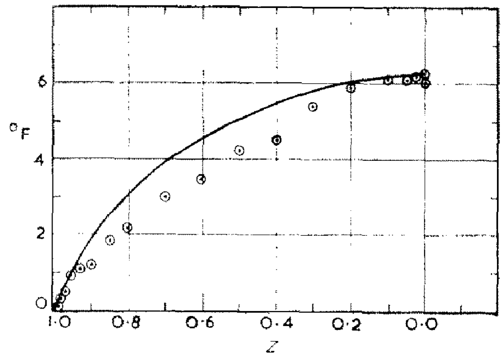

FIG. 6. Comparison of measured temperature distribution in mercury by Isakoff [11] with calculated profile,

○ Isakoff $N_{Re} = 37{\cdot}3 \times 10^4$
— Calculated.

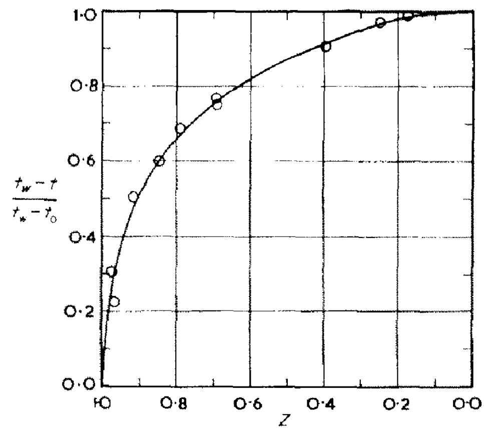

FIG. 7. Normalized temperature profile,

○ Brown *et al.* [14] $N_{Re} = 66 \times 10^4$
— Calculated.

CONCENTRATION PROFILES

Equations (13)–(18) may be used to calculate concentration profiles by replacing the heat transfer groups by the corresponding mass transfer groups. The assumptions made in the previous section concerning the relative effects of eddy and molecular transport differ from those of Lin *et al.* in the region $y^+ > 33$. In this turbulent region the assumptions made become important only at low N_{Pr} and the equations for concentration profiles therefore give results very close to those of Lin *et al.* The agreement of the present equations with experimental results is discussed in a later section.

EVALUATION OF HEAT TRANSFER COEFFICIENTS

Defining the heat transfer coefficient as (see Appendix IV)

$$h = \frac{dq}{dA} \cdot \frac{1}{t_W - t_b} = \frac{q}{A} \cdot \frac{1}{t_W - t_b} \qquad (19)$$

values of h may be obtained if the driving force is known and this is determined from the profile by substituting values for the relevant sections into the heat balance equation:

$$t_W - t_b = \int_0^1 2ZU(t_W - t)\,dZ. \qquad (20)$$

Thus, noting that

$$y = R^+(1 - Z)$$

and

$$dy^+ = -R^+\,dZ$$

and that when

$$y^+ = 0, \qquad Z = 1$$
$$y^+ = R^+, \qquad Z = 0$$
$$y^+ = R^+/5, \qquad Z = 0{\cdot}8$$

we obtain for the five regions of the temperature profile:

$$\left.\begin{aligned} t_W - t_b = &\int_{y^+=5}^{y^+=0} 2UZ(t_W - t)\,dZ + \int_{y^+=33}^{y^+=5} 2UZ(t_W - t)\,dZ \\ &+ \int_{y^+=100}^{y^+=33} 2UZ[(t_W - t_B) + (t_B - t)]\,dZ \\ &+ \int_{y^+=(R^+/5)}^{y^+=100} 2UZ[(t_W - t_B) + (t_B - t_M) + (t_M - t)]\,dZ \\ &+ \int_{y^+=R^+}^{y^+=(R^+/5)} 2UZ[(t_W - t_B) + (t_B - t_M) + (t_M - t)]\,dZ. \end{aligned}\right\} \qquad (21)$$

Noting that $dt = -(\text{const.}/u^*)\,.\,du$, $(t_M - t)$ is replaced by $[q/A\rho Cp(u^*)^2]\,.\,(u - u_M)$, and neglecting the first two terms in equation (21) as negligible, the following form of the equation is obtained:

$$\left.\begin{aligned} t_W - t_b = &(t_W - t_B)\int_{y^+=R^+}^{y^+=33} 2UZ\,dZ + \int_{y^+=100}^{y^+=33} 2UZ(t_B - t)\,dZ \\ &+ (t_B - t_M)\left\{\int_{y^+=(R^+/5)}^{y^+=100} 2UZ\,dZ + \int_0^{0\cdot 8} 2UZ\,dZ\right\} \\ &- \frac{q}{A\rho Cp}\left\{\frac{u_M}{(u^*)^2}\int_{y^+=(R^+/5)}^{y^+=100} 2UZ\,dZ + \frac{u_M}{(u^*)^2}\int_0^{0\cdot 8} 2UZ\,dZ \right. \\ &\left. - \frac{u_b}{(u^*)^2}\int_{y^+=(R^+/5)}^{y^+=100} 2U^2Z\,dZ - \frac{u_b}{(u^*)^2}\int_0^{0\cdot 8} 2U^2Z\,dZ\right\}. \end{aligned}\right\} \qquad (22)$$

The second term of equation (22) has been found to be very small and is omitted from subsequent forms of the equation. Integrating equation (22) [See Appendix II] gives the final form

$$N_{St} = \frac{f/2}{\varphi_H} \qquad (23)$$

and values of φ_H may be calculated from equations (24) and (25) or obtained from Fig. 8. If interpolation between the curves of Fig. 8 at low N_{Pr} is necessary, a plot of $(f/2)/\varphi$ versus N_{Pe} in the required range will give a curve from which the desired values of φ may be obtained for any N_{Re}.

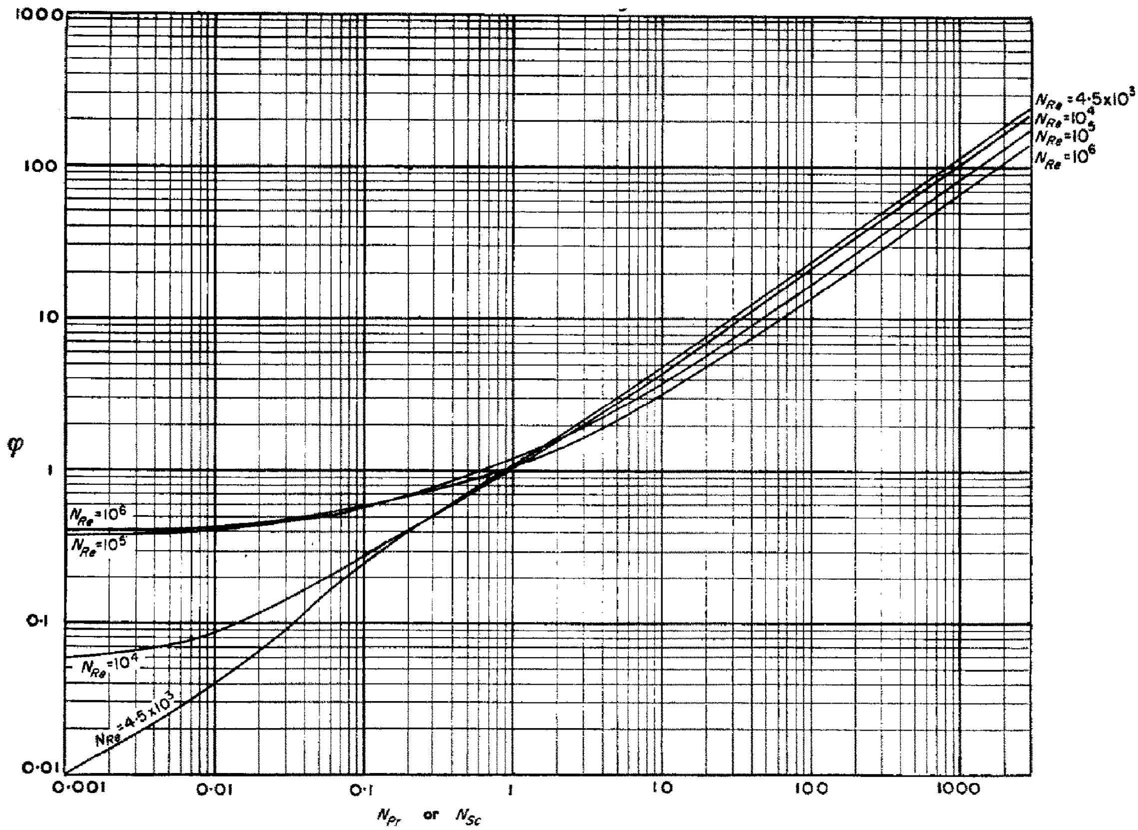

FIG. 8. Plot of φ for use with equations (23) and (28).

(a) For $N_{Re} \geqslant 4 \times 10^4$

$$\begin{aligned}
\varphi_H = {} & \left(\sqrt{\frac{f}{2}}\right)\bigg\{\frac{14\cdot5}{3} N_{Pr}{}^{2/3}\, \tfrac{1}{2} \ln\left[\frac{[1 + N_{Pr}(5/14\cdot5)]^2}{1 - N_{Pr}{}^{1/3}(5/14\cdot5) + N_{Pr}{}^{2/3}(5/14\cdot5)^2}\right] \\
& + (\sqrt{3}) \tan^{-1} \frac{N_{Pr}{}^{1/3}(10/14\cdot5) - 1}{\sqrt{3}} + \frac{(\sqrt{3})\pi}{6} + 5 \ln \frac{1 + 5\cdot64 N_{Pr}}{1 + 0\cdot041 N_{Pr}} \\
& + \left(2\cdot5\, \frac{1 + N_{Pe}[\sqrt{(f/2)}/5]}{2 + N_{Pe}[\sqrt{(f/2)}/5]}\left[\ln \frac{100 + 2\cdot5 N_{Pr}}{33 + 2\cdot5 N_{Pr}}\right] - 17\right)\left(0\cdot64 + 1\cdot24\sqrt{\frac{f}{2}}\right)\bigg\} \\
& + \left(\sqrt{\frac{f}{2}}\right) \cdot \frac{2}{R^+}\bigg\{5\cdot5\left(\frac{R^+}{5} - 100\right) - \frac{5\cdot5}{2R^+}\left(\frac{(R^+)^2}{25} - 10^4\right) \\
& + 2\cdot5\left(\frac{R^+}{5} \ln \frac{R^+}{5} - \frac{R^+}{5} - 360\right) - 2\cdot5\left(\frac{R^+}{50} \ln \frac{R^+}{5} - \frac{R^+}{100} - \frac{2\cdot05 \times 10^4}{R^+}\right)\bigg\} \\
& + \frac{f}{2} \cdot \frac{2}{R^+}\bigg\{30\left(\frac{R^+}{5} - 100\right) - \frac{30}{R^+}\left(\frac{(R^+)^2}{50} - 0\cdot5 \times 10^4\right) \\
& + 13\cdot75\left(\frac{R^+}{5} \ln \frac{R^+}{5} - \frac{R^+}{5} - 360\right) - \frac{13\cdot75}{R^+}\left(\frac{(R^+)^2}{50} \ln \frac{R^+}{5} - \frac{(R^+)^2}{100} - 2 \times 10^4\right)
\end{aligned} \qquad (24)$$

$$+ 6{\cdot}25\left(\frac{R^+}{5}\left[\ln\frac{R^+}{5}\right]^2 - \frac{2R^+}{5}\ln\frac{R^+}{5} + \frac{2R^+}{5} - 1400\right)$$
$$- \frac{6{\cdot}25}{R^+}\left(\frac{(R^+)^2}{50}\left[\ln\frac{R^+}{5}\right]^2 - \frac{(R^+)^2}{50}\ln\frac{R^+}{5} + \frac{(R^+)^2}{100} - 8{\cdot}6\times 10^4\right)\Bigg\}$$
$$+ [0{\cdot}64 + 3{\cdot}8\sqrt{(f/2)} + 18{\cdot}45 f/2]. \qquad (24)$$

(b) For $4500 \leqslant N_{Re} \leqslant 20\,000$, the heat transfer coefficient is again obtained from equation (23) but now

$$\varphi_H = \left(\sqrt{\frac{f}{2}}\right)\left\{\frac{14{\cdot}5}{3} N_{Pr}^{2/3}\,\tfrac{1}{2}\ln\left[\frac{[1 + N_{Pr}(5/14{\cdot}5)]^2}{1 - N_{Pr}^{1/3}(5/14{\cdot}5) + N_{Pr}^{2/3}(5/14{\cdot}5)^2}\right]\right.$$
$$\left. + (\sqrt{3})\tan^{-1}\frac{N_{Pr}^{1/3}(10/14{\cdot}5) - 1}{\sqrt{3}} + \frac{\pi\sqrt{3}}{6} + 5\ln\frac{1 + 5{\cdot}64 N_{Pr}}{1 + 0{\cdot}041 N_{Pr}}\right\}$$
$$+ 2{\cdot}5\left(\sqrt{\frac{f}{2}}\right)\Bigg(\frac{1 + N_{Pe}[\sqrt{(f/2)}/5]}{2 + N_{Pe}[\sqrt{(f/2)}/5]}\cdot\left[\ln\frac{(R^+/5) + 2{\cdot}5N_{Pr}}{33 + 2{\cdot}5N_{Pr}}\right]\left(0{\cdot}64 + 1{\cdot}24\sqrt{\frac{f}{2}}\right)$$
$$- \frac{1 - 0{\cdot}35\sqrt{(f/2)}}{1 + 28{\cdot}8/[N_{Pe}\sqrt{(f/2)}]}\left\{\left[1 + 4{\cdot}25\sqrt{\frac{f}{2}}\right]\left[0{\cdot}64 - \left(1 - \frac{100}{R}\right)^2\right]\right.$$
$$\left. - 3{\cdot}6\sqrt{\frac{f}{2}}\left[0{\cdot}41 - \left(1 - \frac{100}{R^+}\right)^4\right]\right\}\Bigg)$$
$$+ \frac{(1 + 4{\cdot}25\sqrt{f/2})^2\{0{\cdot}64 - [1 - (100/R^+)]^2\} - 7{\cdot}2\,[\sqrt{(f/2)} + 4{\cdot}25(f/2)]\times\{0{\cdot}41 - [1 - (100/R^+)]^4\}}{1 + [28{\cdot}8/N_{Pe}\sqrt{(f/2)}]}$$
$$+ \frac{17{\cdot}28\,(f/2)\{0{\cdot}212 - [1 - (100/R^+)]^6\}}{1 + [28{\cdot}8/N_{Pe}\sqrt{(f/2)}]} - \left(1 - 0{\cdot}35\sqrt{\frac{f}{2}}\right)\left[\left(1 + 4{\cdot}25\sqrt{\frac{f}{2}}\right)\right.$$
$$\left.\times\left(1 - \frac{100}{R^+}\right)^2 - 3{\cdot}6\sqrt{\frac{f}{2}}\left(1 - \frac{100}{R^+}\right)^4\right] + \left\{\left(1 + 4{\cdot}25\sqrt{\frac{f}{2}}\right)^2\left(1 - \frac{100}{R^+}\right)^2\right.$$
$$\left. - 7{\cdot}2\left[\left(\sqrt{\frac{f}{2}}\right) + 4{\cdot}25\frac{f}{2}\right]\left(1 - \frac{100}{R^+}\right)^4 + 17{\cdot}28\frac{f}{2}\left(1 - \frac{100}{R^+}\right)^6\right\}. \qquad (25)$$

The derivation of this equation is outlined in Appendix III.

EVALUATION OF MASS-TRANSFER COEFFICIENTS

Defining the mass-transfer coefficient as

$$k_c = \frac{N}{C_W - C_b} \qquad (26)$$

with

$$C_W - C_b = \int_0^1 2ZU(C_W - C)\,dZ \qquad (27)$$

the same procedure as employed in the previous section gives

$$N_{Sh} = \frac{f/2}{\varphi_D} \qquad (28)$$

where φ_D is obtained from equations (24) and (25) by replacing the heat transfer groups by the relevant mass transfer groups.

The values of φ obtained for smooth pipes from equations (24) and (25) are given in Fig. 8.

COMPARISON OF THEORETICAL AND EXPERIMENTAL DATA

Coefficients of heat and mass transfer have been calculated using equations (23) and (28), and the results compared with other theoretical equations and with experimental data. The most effective comparison of any general theoretical equation is with experimental values at N_{Pr} or N_{Sc} far removed from unity. Agreement with experimental data at high N_{Sc} is evidence for the correctness of the equation in describing transport in the region close to the wall. (For $N_{Sc} > 1000$, 99 per cent of the resistance to transfer is in the region $y^+ < 5$). Similarly, agreement with experimental data at low N_{Pr} indicates correctness of the assumptions made for the transfer across the bulk of the pipe. (In this case about 80 per cent of the resistance is in the region $y^+ > 33$ for a $N_{Pr} = 0{\cdot}01$ and at a $N_{Re} = 10^4$, and 97 per cent at $N_{Re} = 10^6$.) Comparison with experimental data at high values of N_{Sc} has already been made by Lin *et al.*, and the emphasis in the following sections is therefore on low N_{Pr} work, although results are given for other values of N_{Pr} and N_{Sc} where experimental data are available.

Heat Transfer—N_{Pr} from 0·001–0·1

Fig. 9 shows that the present analysis gives excellent agreement with the experimental data in this region. It is interesting to note that the next best agreement is given by Azer and Chao's semi-empirical equation (which is limited to $N_{Pr} < 0{\cdot}1$) and that their equation and equations (23), (24) and (25) include the effect of N_{Pr} in addition to N'_{Pe}. Deissler's analysis has some points of similarity with the present work but gives high results for $N'_{Pe} > 4000$ ($N_{Pr} = 0{\cdot}022$). It is unfortunate that nearly all the experimental work has been carried out at N_{Pr} of 0·020–0·024 so that the effect of N_{Pr} as a variable in addition to N'_{Pe} cannot be satisfactorily assessed.

It will also be seen from Fig. 9 that agreement of the present theory with experimental results is very good down to N'_{Pe} of 400 but that below this, experimental results are lower than predicted. It would be possible to make the present analysis fit the experimental values more closely by reducing the limiting value for the effect of molecular conductivity to below $y^+ = 100$. The value of 100 however is based on available temperature profiles and until more data in the region $N'_{Pe} < 2000$ become available such an adjustment is not warranted.

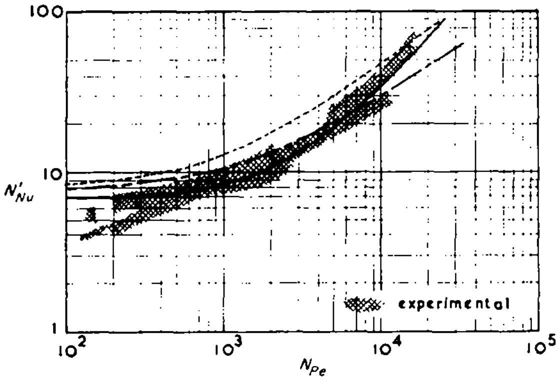

FIG. 9. Comparison of experimental N'_{Nu} for liquid metals [16] with the present analysis.
— — — — — Azer and Chao $N'_{Nu} = 7 + 0{\cdot}05\, N'^{0{\cdot}77}_{Pe}\, N_{Pr}^{0{\cdot}25}$
— — — — — Lyon $N'_{Nu} = 7 + 0{\cdot}025\, N'^{0{\cdot}8}_{Pe}$
———— - - Present analysis, equations (23), (24) and (25).

The most interesting aspect of Fig. 9 is that it shows that the assumptions of the equality of ϵ_H and ϵ_M and of neglecting the conduction from an eddy, which are contrary to the premises of other workers, may very well be correct at low N_{Pr}. As stated above, additional experimental data are most desirable at this stage.

Mass Transfer—N_{Sc} > 1000

Excellent analyses of transport at high N_{Sc} have been made by Deissler and by Lin *et al.* Deissler considers that turbulence continues on a decreasing scale right to the wall, whereas Lin *et al.* follow Rannie's suggestion that eddies exist in the laminar layer. Both ideas will give the same results. The procedure of Lin *et al.* was employed here as it fits in better with the approach used for the remainder of the derivation and the present equations reduce to those of Lin *et al.* in this high N_{Sc} range.

The data of Linton and Sherwood [16] for solution of cinnamic acid at a N_{Sc} of 3000 are shown in Fig. 10. The data can be correlated

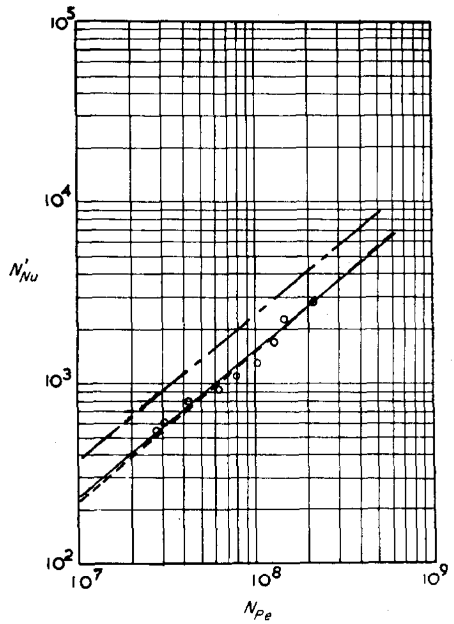

FIG. 10. Comparison of experimental N'_{Nu} for high N_{Sc} with the present analysis,

○ Linton and Sherwood [16] $N_{Sc} = 3000$
— — — — Friend and Metzner
- - - - - - - - Deissler
———— Present analysis.

equally well with the equations of either Deissler or Lin *et al.* The equation of Friend and Metzner [17] gives results approximately 30 per cent too high in this range.

Heat and Mass Transfer—N_{Pr} or N_{Sc} = 0·5–100

From Fig. 11 it will be seen that equations (23) and (28), in common with a number of other analyses, represent the experimental data reasonably well in this intermediate range of N_{Pr} and N_{Sc}. The comparison with experimental results has been shown for three values of N_{Pr} or N_{Sc} (0·6, 10 and 95) at which experimental data are available, and curves representing the analyses of Deissler [7] and of Friend and Metzner [17] are also shown in each case.

The experimental data used for comparison at $N_{Pr} = 95$ are those of Morris and Whitman [18] on the heating and cooling of straw oil ($N_{Pr} = 92$–100) and of Friend and Metzner on heating fluid SD at low Δt ($N_{Pr} = 93$). At $N_{Pr} = 10$ the experimental results used are those of Eagle and Ferguson [19], calculated from their Table II. At N_{Pr} or $N_{Sc} = 0{\cdot}6$ the mass transfer data of Jackson and Ceaglske [20] for the vaporization of water into air and the heat transfer data of Colborn and Coghlan [21] for

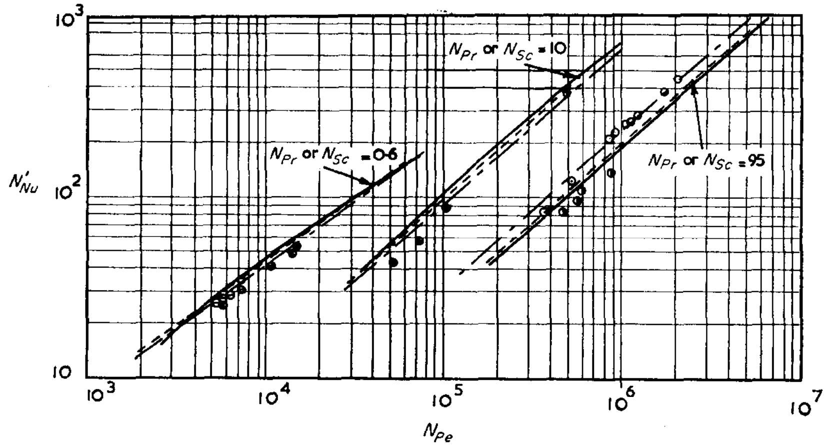

FIG. 11. Comparison of experimental N'_{Nu} for intermediate N_{Sc} and N_{Pr} with the present analysis,

○ Friend and Metzner [17] $N_{Pr} = 93$
◑ Morris and Whitman [18] $N_{Pr} = 92$–100
⊕ Eagle and Ferguson [19] $N_{Pr} = 10$
⊗ Jackson and Ceaglske [20] $N_{Sc} = 0{\cdot}6$
⊖ Colburn and Coghlan [21] $N_{Sc} = 0{\cdot}48 - 0{\cdot}75$
— — — — Friend and Metzner
- - - - - - - - Deissler
———— Present analysis.

heating N_2/H_2 mixtures ($N_{Pr} = 0{\cdot}48$–$0{\cdot}75$) were used. This latter range (N_{Pr} or $N_{Sc} = 0{\cdot}6$) is the lower limit of applicability of Friend and Metzner's equation.

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APPENDIX I

In evaluating the ratio $f(Z)_H/f(Z)_M$ the following points have been considered:

(i) Azer and Chao [5] have shown that

$$f(Z)_H = \frac{\int_0^Z 2UZ\,dZ}{Z}$$

for constant flux. They have evaluated this function using a logarithmic velocity distribution and shown that it may be approximately represented by $Z^{0{\cdot}75}$.

Another relationship for $f(Z)_H$ can be evaluated [8] as

$$f(Z)_H = Z[1 + 4{\cdot}25\sqrt{(f/2)} - 3{\cdot}6\sqrt{(f/2)}\,Z^2]$$

for a parabolic velocity distribution in the central core and this can be approximated by $1{\cdot}1Z$.

Because $1{\cdot}1Z$ gives values very close to $Z^{0{\cdot}75}$ for $f(Z)_H$ for the N_{Re} concerned and because the profile equation was already complex, it was decided that further elaboration to make allowance for the small departure of $f(Z)_H$ from Z was undesirable. $f(Z)_H$ was therefore assumed equal to Z. It is probably this approximation which causes the values of φ in Fig. 8 to differ slightly from unity at N_{Pr} or $N_{Sc} = 1$.

(ii) The usual momentum transfer relation $f(Z)_M = Z$, which follows from the fact that $\tau_r/\tau_s = r/R = Z$, then gives the result that $f(Z)_H/f(Z)_M = 1$ which is used in obtaining equation (12). (As stated the approximation is substantiated by the data of Figs. 1 and 2.)

APPENDIX II

The heat transfer equations (23 and 24) for $Re \geqslant 20\,000$ may be obtained by substituting the expressions given below in equation (22).

(i) $(t_W - t_B)$

$$= \frac{q}{A\rho Cp} \cdot \frac{1}{u^*} \left\{ \frac{14\cdot5}{3} N_{Pr}^{2/3} \times \tfrac{1}{2} \ln \frac{[1 + N_{Pr}(5/14\cdot5)]^2}{1 - N_{Pr}^{1/3}(5/14\cdot5) + N_{Pr}^{2/3}(5/14\cdot5)^2} + (\sqrt{3}) \tan^{-1} \left[\frac{N_{Pr}^{1/3}(10/14\cdot5) - 1}{\sqrt{3}} \right] + \frac{\pi\sqrt{3}}{6} + 5 \ln \frac{1 + 5\cdot64 N_{Pr}}{1 + 0\cdot041 N_{Pr}} \right\}$$

from equations (13, 14)

(ii) $$\int_{y^+=R^+}^{y^+=33} 2UZ\,dZ \doteqdot \int_0^1 2UZ\,dZ = 1.$$

Where

$$y^+ = R^+ \text{ at } Z = 0$$

$$y^+ = 33 \text{ at } Z = 1 - \frac{33}{R^+} \doteqdot 1.$$

(iii) $$\int_{y^+=(R^+/5)}^{y^+=100} 2UZ\,dZ$$

$$= \frac{2\sqrt{(f/2)}}{R^+} \int_{100}^{R^+/5} \left(1 - \frac{y^+}{R^+}\right)(5\cdot5 + 2\cdot5 \ln y^+)\, dy^+$$

$$= \frac{2\sqrt{(f/2)}}{R^+} \left[5\cdot5\, y^+ - \frac{5\cdot5(y^+)^2}{2R^+} + 2\cdot5\,(y^+ \ln y^+ - y^+) - \frac{2\cdot5}{R^+} \left(\frac{[y^+]^2}{2} \ln y + - \frac{[y^+]^2}{4} \right) \right]_{100}^{R^+/5}$$

Where

$$Z = 1 - y^+/R^+, \quad dZ = -\frac{dy^+}{R^+}$$

$$U = \sqrt{(f/2)}\,(5\cdot5 + 2\cdot5 \ln y^+)$$

(iv) $t_B - t_M$

$$= \frac{q}{A\rho Cp} \cdot \frac{2\cdot5}{u^*} \left\{ \frac{1 + N_{Pe}[\sqrt{(f/2)}/5]}{2 + N_{Pe}[\sqrt{(f/2)}/5]} \right\} \times \ln \frac{100 + (2\cdot5/N_{Pr})}{33 + (2\cdot5/N_{Pr})}.$$

from equation (16)

(v) $U_B = \sqrt{(f/2)}\,(5\cdot5 + 2\cdot5 \ln 100) = 17\sqrt{(f/2)}$

(vi) $$\int_0^{0\cdot8} 2UZ\,dZ$$

$$= \int_0^{0\cdot8} 2Z \left[\frac{u_0}{u_b} - 7\cdot2\sqrt{(f/2)}\,Z^2 \right] dZ$$

$$= \left[\frac{u_0}{u_b} Z^2 - 3\cdot6\sqrt{(f/2)}\,Z^4 \right]_0^{0\cdot8} + 0\cdot64 + 1\cdot24\sqrt{(f/2)}$$

where

$$U = \frac{u_0}{u_b} - 7\cdot2\sqrt{(f/2)}\,Z^2$$

and

$$\frac{u_0}{u_b} = 1 + 4\cdot25\sqrt{(f/2)}.$$

(vii) $$\int_{y^+=(R^+/5)}^{y^+=100} 2U^2Z\,dZ$$

$$= \frac{f}{R^+} \int_{100}^{R^+/5} [5\cdot5 + 2\cdot5 \ln y^+]^2 \left[1 - \frac{y^+}{R^+}\right] dy^+$$

$$= \frac{f}{R^+} \left[30y^+ - \frac{15(y^+)^2}{R^+} + 13\cdot75\,[y^+ \ln y^+ - y^+] - \frac{1\cdot375}{R^+} \left\{ \frac{(y^+)^2}{2} \ln y + - \frac{(y^+)^2}{4} \right\} + 6\cdot25 \left\{ \frac{(y^+)^2}{2} (\ln y^+)^2 - 2y^+ \ln y^+ + 2y^+ \right\} - \frac{6\cdot25}{R^+} \left\{ \frac{(y^+)^2}{2} (\ln y^+)^2 - \frac{(y^+)^2}{2} \ln y^+ \right\} + \frac{(y^+)^2}{4} \right]_{100}^{R^+/5}$$

(viii) $$\int_0^{0\cdot8} 2U^2Z\,dZ$$

$$= \int_0^{0\cdot8} 2 \left[\frac{u_0}{u_b} - 7\cdot2\sqrt{(f/2)}\,Z^2 \right]^2 Z\,dZ$$

$$= \left[\left(\frac{u_0}{u_b}\right)^2 Z^2 - 7\cdot2\sqrt{(f/2)}\,\frac{u_0}{u_b} Z^4 + 8\cdot64 f Z^6 \right]_0^{0\cdot8}$$

where

$$\frac{u_0}{u_b} = 1 + 4{\cdot}25\,\sqrt{(f/2)}.$$

By substituting these values in equation (22) and taking out as common factor

$$\frac{q}{A\rho Cp}\,\frac{1}{u_b}\cdot\frac{1}{f/2}$$

the final equation $N_{St} = (f/2)/\varphi_H$ is obtained.

APPENDIX III

For the range $20\,000 > N_{Re} \geqslant 4500$, $R^+/5 < 100$ so that the logarithmic regions disappear completely and the following equation replaces equations (22)

$$\begin{aligned}(t_W - t_b) &= \int_{y^+=33}^{y^+=0} 2UZ\,(t_W - t)\,\mathrm{d}Z + (t_W - t_B)\\ &\times \int_{y^+=R^+}^{y^+=33} 2UZ\,\mathrm{d}Z\\ &+ \int_{R^+/5}^{y^+33} 2UZ\,(t_B - t)\,\mathrm{d}Z + (t_B - t_C)\\ &\times \int_{R^+}^{R^+/5} 2UZ\,\mathrm{d}Z\\ &+ \int_{100}^{R^+/5} 2UZ\,(t_C - t)\,\mathrm{d}Z\\ &+ \int_{R^+}^{100} 2UZ\,(t_M - t)\;\mathrm{d}Z + (t_C - t_M)\\ &\times \int_{R^+}^{100} 2UZ\,\mathrm{d}Z.\end{aligned}$$

As the first term is no longer negligible, the first two terms together are taken as equal to $(t_W - t_B)$ to give the same first term as in equation (22). The region $33 \geqslant y^+ \geqslant 100$ still has a negligible effect. The fourth term is evaluated as in Appendix II, but it should be noted that this term only exists if $R^+/5 > 33$. The fifth, sixth and seventh terms are evaluated by substituting the relevant functions given below.

$$(t_C - t) = \frac{q}{A\rho Cp}\cdot\frac{1}{(u^*)^2}\,(u - u_C)\,\frac{1}{1 + (a/\epsilon)}$$

and in the central parabolic region

$$\epsilon_M = Ru^*/14{\cdot}4$$

$$\therefore\quad a/\epsilon_M = 28{\cdot}8/[N_{Pe}(f/2)]$$

$$t_M - t = \frac{q}{A\rho Cp}\cdot\frac{1}{(u^*)^2}\,(u - u_M)$$

$$\begin{aligned}t_C - t_M &= \frac{q}{A\rho Cp}\cdot\frac{1}{(u^*)^2}\,(u_M - u_C)\cdot\frac{1}{1 + 28{\cdot}8/[N_{Pe}(f/2)]}\\ &= \frac{q}{A\rho Cp}\cdot\frac{1}{(u^*)^2}\,(7{\cdot}2u^*)\left[0{\cdot}64 - \left(1 - \frac{100}{R^+}\right)^2\right]\end{aligned}$$

and the remainder of these terms was evaluated as in Appendix II.

APPENDIX IV

The requirement of constant flux was made because a heat balance to any distance Z from the centre of the tube gives for equation (2)

$$(\epsilon_H + a)\,\frac{\partial t}{\partial y} = \frac{q}{A}\cdot\frac{1}{\rho Cp}\cdot\frac{\int_0^z 2UZ\,\mathrm{d}Z}{Z} \qquad (29)$$

for constant flux and

$$(\epsilon_H + a)\,\frac{\partial t}{\partial y} = \frac{\partial q}{\partial A}\cdot\frac{1}{\rho Cp}\cdot\frac{\int_0^z 2UZ\,(t_W - t)\,\mathrm{d}Z}{Z(t_W - t_M)} \qquad (30)$$

for constant wall temperature.

Therefore only for constant flux is $\partial q/\partial A = q/A$ and also equation (30) cannot be solved directly as the variables are not separable.

This is not a serious limitation as heat transfer coefficients for constant flux and constant wall temperature are the same for $N_{Pr} > 0{\cdot}5$ [5]. For lower values of N_{Pr} theoretical considerations [4, 6] have indicated that the transfer coefficients differ slightly in the two cases but experimental confirmation is inadequate [6].

Résumé—Les auteurs proposent un modèle simple pour le transport de masse ou de chaleur, fondé sur une forme modifiée de l'analogie de Reynolds. Ils établissent des équations permettant de calculer, pour n'importe quelle valeur de N_{Pr} ou N_{Sc}, les coefficients de transport de masse et de chaleur et les profils de température et de concentration. Pour établir ces équations, ils supposent: 1° que, dans la conduite, le mécanisme de transport est tel qu'il n'y a pas de transport moléculaire appréciable dans le noyau turbulent, même aux bas nombres de Prandtl et 2° que le transport tourbillonnaire est uniquement fonction du type d'écoulement. L'équation est de la forme

$$N_{St}\text{ (ou } N_{Sh}) = \frac{f/2}{\varphi}$$

φ est donné à la fois sous forme analytique et sous forme graphique en fonction de N_{Pr} (ou N_{Sc}), pour des conduites lisses.

Les calculs de N'_{Nu} et les profils de températures sont en bon accord avec les données expérimentales. Une attention particulière est portée sur les résultats aux bas nombres de Prandtl où les hypothèses faites quant au transport dans le noyau turbulent sont les plus valables; dans cette région, les équations proposées sont mieux vérifiées par les résultats expérimentaux que les autres relations. Dans le domaine intermédiaire de nombres de Prandtl ou de Schmidt, les équations proposées sont en bon accord avec les autres analyses et pour les valeurs élevées de N_{Sc}, les équations se réduisent à celles de Lin et autres [1] qui sont en excellent accord avec les données expérimentales.

Zusammenfassung—Für den turbulenten Wärme- und Stofftransport wird mit einer modifizierten Form der Reynoldsanalogie ein einfaches Modell vorgeschlagen. Danach abgeleitete Gleichungen liefern Wärme- bzw. Stoffübergangskoeffizienten und Temperatur- bzw. Konzentrationsprofile für beliebige Werte von *Pr* bzw. *Sc*. In der Ableitung wird angenommen:

(1) der Transportmechanismus der Rohrströmung verhält sich so, dass kein wesentlicher Molekulartransport im turbulenten Kern auftritt, selbst bei kleinen *Pr*;
(2) der Wirbeltransport ist nur eine Funktion des Strömungsprofils.

Die Gleichung hat die Form

$$St\text{ (bzw. } Sh) = \frac{f/2}{\varphi}$$

wobei φ sowohl algebraisch wie auch als Kurve φ über *Pr* (bzw. *Sc*) für glatte Rohre gegeben ist.

Berechnungen von Nu' und Temperaturprofilen stimmen gut mit Versuchswerten überein. Besondere Sorgfalt wurde zur Berechnung bei kleinen *Pr* aufgewendet, da hier obige Annahmen für den Transport im turbulenten Kern von grösstem Einfluss sind. In diesem Bereich geben die vorgeschlagenen Gleichungen besser als andere Korrelationen experimentelle Ergebnisse wieder. In einem Zwischenbereich von *Pr* bzw. *Sc* kongruieren die Gleichungen gut mit anderen Analysen; bei hohen Werten von *Sc* reduzieren sich die Gleichungen auf jene von Lin und anderen [1], die mit Versuchswerten bestens übereinstimmen.

Аннотация—Предлагается простая модель турбулентного тепло-или массообмена, основанная на модифицированной аналогии Рейнольдса. С помощью выведенных уравнений можно определить коэффициенты тепло-и массообмена, профили температуры и концентрации при любом значении N_{Pr} или N_{Sc}. При выводе уравнения предполагается, что в турбулентном ядре потока не происходит значительного молекулярного переноса даже при небольшом значении N_{Pr}, и что перенос вихрей представляет собой функцию только картины потока. Уравнение имеет следующий вид:

$$N_{St}\text{ (или } N_{Sh}) = \frac{f/2}{\varphi}$$

где φ определяется как алгебраически, так и построением графической зависимости φ от N_{Pr} (или N_{Sc}) для гладких труб.

Вычисления N'_{Nu} и температурных профилей хорошо согласуются с экспериментальными данными. Большое внимание уделяется результатам при небольшом значении N_{Pr} в предположении, что перенос в турбулентном ядре имеет наибольший эффект. В этой области экспериментальные результаты более точно определяются предложенными уравнениями, чем другими соотношениями. В промежуточном диапазоне N_{Pr} или N_{Sc} предложенные уравнения согласуются с другими анализами, а при больших значениях N_{Sc} эти уравнения сводятся к уравнениям Лина и других [1], которые находятся в полном соответствии с экспериментальными данными.